Laws of Logarithms

① $\log_a (P \times Q)$

$= \log_a PQ$

$= \log_a P + \log_a Q$

② $\log_a (x \div y)$

$= \log_a \left(\frac{x}{y}\right)$

$= \log_a x - \log_a y$

\# Note: Whenever the base of a logarithms is not given, the base of such logarithms is always taken as 10.

$\log (PQR) = \log P + \log Q + \log R$

$\log (xywz) = \log x + \log y + \log w + \log z$

$\log \left(\frac{xy}{wz}\right) = \log (xy) - \log (wz)$

$= \log x + \log y - \log w - \log z$

$\log \left(\frac{PQR}{TW}\right) = \log (PQR) - \log (TW)$

$= \log P + \log Q + \log R - \log T - \log W$

$\log \left(\frac{y}{abc}\right) = \log y - \log abc$

$= \log y - \log a - \log b - \log c$

Evaluate: $\log 2 + \log 7 + \log 3 - \log 9$

Solution

$\log 2 + \log 7 + \log 3 - \log 9$

$= \log \left(\frac{7 \times 3}{2 \times 9}\right)$

$= \log 7/6$

\# Two logs are said to be alike if they have the same base and the same number.

$8 \log_5 4 + 7 \log_5 4 = 15 \log_5 4$

$8 \log_2 3 + 6 \log_2 3 = 14 \log_2 3$

$7 \log_3 5 - 4 \log_3 5 = 3 \log_3 5$

③ Special logarithm

$\log_2 2 = 1$

$\log_7 7 = 1$

$\log_a a = 1$

Whenever the base of algorithms is the same as the number. The logarithms of number is always equal to 1.

④ $\log_a P = \frac{\log P}{\log a}$

If they have the same as P

$\log_a P = \frac{\log_P P}{\log_P a}$

$= \frac{1}{\log_P a}$

Without using tables, simplify

$\log_4 2 + \log_4 32 - \log_4 0.25$

<u>Solution</u>

$\log_4 2 + \log_4 32 - \log_4 0.25$

$= \log_4 \left(\frac{2 \times 32}{0.25}\right)$

$= \log_4 \frac{64}{0.25}$

$= \log_4 256$

$= \log_4 4^4$

$= 4 \log_4 4$

$= 4 \times 1$

$= 4$

Without using tables, simplify

$\log_2 4^{x+1} = 4$

<u>Solution</u>

$\log_2 4^{x+1} = 4 \log_2 2$

$\log_2 4^{x+1} = \log_2 2^4$

$\log_2 4^{x+1} = \log_2 2^4$

$\log_2 4^{x+1} = \log_2 16$

$4^{x+1} = 16$

$4^{x+1} = 4^2$

$x + 1 = 2$

$x = 1$

Without using tables, simplify

$\log x = 3 \log 1.2 + 2 \log \left(\frac{1}{3}\sqrt{10}\right) - \log_{10} 0.96$

<u>Solution</u>

$\log_{10} x = \log_{10} 1.2^3 + \log_{10} \left(\frac{1}{3}\sqrt{10}\right)^2 - \log_{10} 0.96$

$\log_{10} x = \log_{10} 1.728 + \log_{10} \frac{10}{9} - \log_{10} 0.96$

$\log_{10} x = \log_{10} \left(1.728 \times \frac{10}{9} \times \frac{1}{0.96}\right)$

$\log_{10} x = \log_{10} 2$

$x = 2$

Without using tables, simplify

$\log_9 3 + \log_9 243 - 2 \log_9 3$

<u>Solution</u>

$\log_9 3 + \log_9 243 - \log_9 3^2$

$\log_9 3 + \log_9 243 - \log_9 9$

$\log_9 \left(\frac{3 \times 243}{9}\right)$

$= \log_9 81$

$= \log_9 9^2$

$= 2 \log_9 9$

$= 2 \times 1$

$= 2$

Second way [illegible] simplify

$\log_{10} x + \log_{10} y = 4$

$\log_{10} x + \log_{10} y^2 = 3$

<u>Solution</u>

$\log_{10} x + \log_{10} y = 4\log_{10} 10$

$\log_{10} x + \log_{10} y = \log_{10} 10^4$

$\log_{10} xy = \log_{10} 10000$

$xy = 10000$ — (i)

$\log_{10} x + \log_{10} y^2 = 3$

$\log_{10} x + \log_{10} y^2 = \log_{10} 10^3$

$\log_{10} xy^2 = \log_{10} 1000$

$xy^2 = 1000$ — (ii)

From equation (i) $xy = 10000$
substitute in equation (ii)

$xy^2 = 1000$

$(xy)y = 1000$

$10000y = 1000$

$y = \frac{1000}{10000}$

$y = 1/10$

From equation (i) $xy = 10000$

$x\left(\frac{1}{10}\right) = 10000$

$\frac{x}{10} = 10000$

$x = 100000$

Find the value of x which satisfies the equation

$\log_3 x + \log_x 3 = \frac{10}{3}$

<u>Solution</u>

$\frac{\log x}{\log 3} + \log_x 3 = \frac{10}{3}$

$\frac{\log_x x}{\log_x 3} + \log_x 3 = \frac{10}{3}$

let $\log_x 3 = p$

$\frac{1}{\log_x 3} + \log_x 3 = \frac{10}{3}$

$\frac{1}{p} + p = \frac{10}{3}$

multiply through by $3p$

$3 + 3p^2 = 10p$

$3p^2 - 10p + 3 = 0$

$3p^2 - 9p - p + 3 = 0$

$3p(p-3) - 1(p-3) = 0$

$(3p-1)(p-3) = 0$

$3p - 1 = 0$ or $p - 3 = 0$

$3p = 1$ or $p = 3$

$p = 1/3$ or $p = 3$

$\log_x 3 = 1/3$

$\log_x 3 = 1/3 \log_x x$

$\log_x 3 = \log_x x^{1/3}$

$x^{1/3} = 3$

$\sqrt[3]{x} = 3$

$x = 27$

$\log_x 3 = 3$

$\log_x 3 = 3\log_x x$

$\log_x 3 = \log_x x^3$

$x^3 = 3$

$x = \sqrt[3]{3}$

(5) $a^{\log_a x} = x$

Without using tables, Evaluate

$2^{-2\log_2 3} \times 3^{3\log_3 2}$

Sol

$2^{-2\log_2 3} = x$

$\log_2 2^{-2\log_2 3} = \log_2 x$

[Add log of base 2 to both sides]

$-2\log_2 3 = \log_2 x$

$\log_2 3^{-2} = \log_2 x$

$\log_2 \frac{1}{3^2} = \log_2 x$

$\log_2 \frac{1}{9} = \log_2 x$

$x = \frac{1}{9}$

$3^{3\log_3 2} = y$

[Add log of base 3 to both sides]

$\log_3 3^{3\log_3 2} = \log_3 y$

$3\log_3 2 = \log_3 y$

$\log_3 2^3 = \log_3 y$

$\log_3 8 = \log_3 y$

$y = 8$

$x \times y = \frac{1}{9} \times 8 = \frac{8}{9}$

Without using tables, Evaluate

$5^{2\log_5 3} \div 4^{-2\log_4 3}$

solution

$5^{2\log_5 3} = x$

[Add log of base 5 to both sides]

$\log_5 5^{2\log_5 3} = \log_5 x$

$2\log_5 3 = \log_5 x$ [$\log_5 5 = 1$]

$\log_5 3^2 = \log_5 x$

$\log_5 9 = \log_5 x$

$x = 9$

$4^{-2\log_4 3} = y$

[Add log of base 4 to both sides]

$\log_4 4^{-2\log_4 3} = \log_4 y$

$-2\log_4 3 = \log_4 y$ [$\log_4 4 = 1$]

$\log_4 3^{-2} = \log_4 y$

$\log_4 \frac{1}{3^2} = \log_4 y$

$\log_4 \frac{1}{9} = \log_4 y$

$y = \frac{1}{9}$

$x \div y = 9 \div \frac{1}{9}$

$= 9 \div \frac{1}{9}$

$= 9 \times 9$

$= 81$