**Second Order Differential Equation**

y=Aekx

Differentiate

Substitute

Divide through by

**Real & Different Roots**

Example: Evaluate

***Solution***

Divide through by

K2 + 3K + 2 = 0

K2 + 2K + K + 2 = 0

K(K+2) + 1(K+2) = 0

(K+1) (K+2) = 0

K = –1 or K = –2

Substitute for K respectively

**Example: Evaluate**

Divide through by

K2 – 7K + 12 = 0

Factorize,

K2 – 7k + 12 = 0

 12K2

–4K –3K

K2 – 4K – 3K + 12 = 0

K(K – 4) – 3(K – 4) = 0

(K – 3) (K – 4) = 0

K = 3 or K = 4

Example: Evaluate

*Solution*

K2 + 3k – 10 = 0

 - 10K2

5K –2K

K2 + 5K – 2K – 10 = 0

K(K+5) – 2(K+5) = 0

(K – 2) (K+5) = 0

K = 2 or K = –5

**Real & Equal Roots to the Auxilliary Equation**

K2 + 6k + 9 = 0

 9K2

3K 3K

K2 + 3K + 3K + 9 = 0

K(K+3) +3(K+3) = 0

(K+3) (K+3) = 0

K= –3 twice

**Complex Roots to Auxilliary Equation**

Suppose K = ± jB

K1 = + jB

K2 = – jB

Substitute K1 and K2 respectively

Note:

**Factorize**

Let A = C+D, B = C – D

**Example:** M = 5 j2, Evaluate

**Solution**

**Example:** Evaluate

**Solution**

K2 + 4K + 9 = 0

Using quadratic formula

Example: Evaluate

Solution

K2 + n2 = 0

K2 = – n2

K = ±jn

Example: Evaluate

Solution

K2 – n2 = 0

K2 = n2

K = ±n

Y = A cos hnx + B sin hnx

Example: Evaluate

Solution

K2 = –16

K2 = ±j4

Y = A cos 4x + B sin 4x

Example: Evaluate

Solution

m2 – 3 = 0

m2 = 3

m =

y = A cos h + B sin h

Example: Evaluate

Solution

K2 + 5 = 0

K2 = –5

K =

K =

Y = A cos + B sin

Example: Evaluate

Solution

K2 – 4 = 0

K2 = 4

K = ±2

Y = A cos hnx + B cos hnx

= A cos h2x + B sin h2x

Example: Evaluate

Solution

K2 – 12k + 36 = 0

 36K2

–6K –6K

K2 – 6K – 6K + 36 = 0

K (K – 6) – 6 (K – 6) = 0

(K – 6) (K – 6) = 0

K = +6 or K = 6

Example: Evaluate

Solution

K2 + 2k – 3 = 0

 –3K2

3K –K

K2 + 3K – K – 3 = 0

K (K + 3) – 1 (K + 3) = 0

(K – 1) (K + 3) = 0

K = 1 or K = –3

Example: Evaluate

Solution

K2 – 9 = 0

K2 = 9

K = ±3

Y = A cos h3x + B sin h3x

Example: Evaluate

Solution

2K2 + 4K + 3 = 0

Factorize

If

 - Complementary function

Y = X – Particular Integral

Example: Solve

Solution

K2 –5x + 6 = 0

 6K2

–2K –3K

K (K – 2) – 3 (K – 2) = 0

(K – 3) (K – 2) = 0

K = 3 or K = 2

 - Complementary function

To find the particular integral we assume the general form of the Right Hand Side (R.H.S) which is a second degree function.

Y = Cx2 + Dx + E

Differentiate,

Substitute in the equation

2c – 5 (2cx + D) + 6 (cx2 + Dx + E) = x2

2c – 10cx – 5D + 6cx2 + 6Dx + 6E = x2

Equating,

6cx2 = x2

c =

–10cx + 6Dx = 0

Substitute,

–10cx + 6Dx = 0x

–10c +6D = 0

–10( ) + 6D = 0

6D =

D =

D =

2c – 5D + 6E = 0

2( ) – 5( ) + 6E = 0

General solution = Complementary function + Particular Integral

|  |  |
| --- | --- |
| **Complementary Function** | **Particular Integral** |
| If f(x) = kf(x)=kxf(x)=kx2f(x)=K sin x “or” K cos xf(x) = K sin hx “or” K cos hxf(x) = | Assuming *Y = C*Y = Cx + DY = Cx2 + Dx + EY = C cos x + D sin xY = C cos hx + D sin hxY =  |

Example: Solve

Solution

K2 –5x + 6 = 0

 6K2

–2K –3K

K2 – 2K – 3K + 6 = 0

K (K – 2) – 3 (K – 2) = 0

(K – 3) (K – 2) = 0

K = 3 or K = 2

 - Complementary function

F(x) = 24

0 – 5(0) + 6C = 24

6C = 24

C=4

General Solution = CF + PI

Example: Solve

Solution

K2 –5x + 6 = 0

 6K2

–2K –3K

K2 – 2K – 3K + 6 = 0

K (K – 2) – 3 (K – 2) = 0

(K – 3) (K – 2) = 0

K = 3 or K = 2

The general form of PI will be

Y = C cos 4x + D sin 4x

sin 4x + 4D cos 4x

Substitute in one given equation

–16C cos 4x – 16D sin 4x – 5 (–4C sin 4x + 4D cos 4x) +6 (C cos 4x + D sin 4x) = 2 sin 4x

= –16C cos 4x – 16 D sin 4x + 20C sin 4x – 20D cos 4x + 6C cos 4x + 6D sin 4x = 2 sin 4x

Collect like terms

–10C cos 4x – 20D cos 4x = 0

–10C – 20D = 0 ------- (i)

–10D sin 4x + 20C sin 4x = 2 sin 4x

–10D + 20C = 2

20C – 10D = 2 -------- (ii)

–10C – 20D = 0 ------- (i) x 2

20C – 10D = 2 --------- (ii) x 1

Adding together @ next step

–20C – 40D = 0

20C – 10D = 2

Adding together

–50D = 2

D = –1/25

20C – 10D = 2

20C – 10 ( = 2

20C = 2

20C =

C =

C=

Example: Solve

Solution

K2 – 14K + 49 = 0

 49K2

7K 7K

K2 + 7K + 7K +49 = 0

K (K + 7) + 7 (K + 7) = 0

(K + 7) (K + 7)

K = –7

The general form of PI will be

Example: Solve

Solution

K2 + 6K + 10 = 0

= – 3 ± j

The general form will be

–4C cos 2x – 4D sin 2x + 6 (–2C sin 2x + 2D cos 2x) + 10 (C cos 2x + D sin 2x) = 2 sin 2x

–4C cos 2x – 4D sin 2x – 12C sin 2x + 12D cos 2x + 10C cos 2x + 10D sin 2x = 2 sin 2x

6C + 12D = 0 ------- (i)

6D sin 2x – 12C sin 2x = 2 sin 2x

6D – 12C = 2

–12C + 6D = 2 --------- (ii)

6C + 12D = 0 ----- (i) X 2

–12C + 6D = 2 ------ (ii) X 1

12C + 24D = 0

–12C + 6D = 2

30D = 2

D = 2/30

D = 1/15

Substitute D = 1/15 in equation (ii)

–12C + 6(1/15) = 2

–12C + 2/5 = 2

–12C = 2 – 2/5

–12C = 8/5

C =

C =

Example: Solve

Solution

K2 – 3K + 2 = 0

K2 – 2K – K + 2 = 0

K (K – 2) – 1 (K – 2) = 0

(K – 1) (K – 2) = 0

K = 1 or K = 2

The general form of PI will be

2C – 3 (2Cx +D) +2 (=

2C – 6Cx – 3D + 2Cx2 + 2Dx + 2E = x2

Equating,

2Cx2 = x2

2C = 1

C = ½

–6Cx + 2Dx = 0

–6C + 2D = 0

–6 (1/2) + 2D = 0

–3 + 2D = 0

2D = 3

D = 3/2

2C – 3D + 2E = 0

2(1/2) – 3 (3/2) + 2E = 0

1 – 9/2 + 2E = 0

–7/2 + 2E = 0

2E = 7/2

E = 7/4

Substitute

X = 0

A + B = –1 ------- (i)

Substitute *x = 0,*

A + 2B = 1 ----- (ii)

A + B = –1

A + 2B = 1

–B = –2

B = 2

A + B = –1

A + 2 = –1

A = –1 – 2

A = –3

Example: Solve the equation

Given that when X = 0, Y = 5/2,

Solution

K2 + 4K + 5 = 0

Using quadratic formula

= –2±j

9Ce3x + 4 (3Ce3x) + 5Ce3x = 13e3x

9Ce3x + 12Ce3x + 5Ce3x = 13e3x

26Ce3x = 13e3x

C = ½

When x = 0, y = ½

Y = A cos 0 + ½

When x = 0,

–1 + 4 = B

B = 3

Example: Solve

Solution

K2 – 2K – 8 = 0

 –8K2

– 4K 2K

K2 – 4K + 2K – 8 = 0

K (K – 4) + 2 (K – 4) = 0

K = –2 or K = 4

To find the particular integral

Substituting in equation

Equating,

PI,

Example: Solve

Solution

K2 + K – 2 = 0

 –2K2

2K –K

K2 + 2K – K – 2 = 0

K (K + 2) – 1 (K – 2) = 0

(K – 1) (K + 2) = 0

K = 1 or K = 2

To find particular Integral

substituting

Example: Solve

Solution

K2 – K – 2 = 0

 –2K2

–2K K

K2 – 2K + K – 2 = 0

K (K – 2) + 1 (K – 2) = 0

(K + 1) (K – 2) = 0

K = –1 or K = 2

To find particular integral

Substituting in equation

–2y = 8

= –4

Example: Solve

Solution

K2 – 4 = 0

K = ±2

Y = A cos h2x + B sin h2x

To find particular integral

Y = A cos h2x + B sin h2x + 2

Example: Solve the equation

Solution

K2 + 2K + 1 = 0

K2 + K + K + 1 = 0

K (K + 1) + 1 (K + 1) = 0

(K + 1) (K + 1) = 0

K = –1

Example: Solve the equation

K2 + 25 = 0

K2 = –25

=±j5

To find particular integral

2C + 25 (Cx2 + Dx + E) = 5x2 + x

2C + 25Cx2 + 25Dx + 25E = 5x2 + x

Equating,

25Cx2 = 5x2

25Dx = x

2C + 25E = 0